A New Equation of Motion for a Particle in Classical Mechanics

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Abstract

This work presents a new equation of motion for a particle in classical mechanics, which can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces.

Equation of Motion

The acceleration $\mathbf{a}_\mathrm{A}$ of a particle A relative to a reference frame S (non-rotating) fixed to a particle S is given by the following equation:

$$\mathbf{a}_\mathrm{A} = \frac{\sum \mathbf{F}_\mathrm{A}}{m_\mathrm{A}} - \frac{\sum \mathbf{F}_\mathrm{S}}{m_\mathrm{S}}$$

where $\sum \mathbf{F}_\mathrm{A}$ is the sum of the forces acting on particle A, m_A is the mass of particle A, $\sum \mathbf{F}_\mathrm{S}$ is the sum of the forces acting on particle S, and m_S is the mass of particle S.

Observations

In contradiction with Newton's first and second laws, from the last equation above it follows that the acceleration of particle A relative to the reference frame S fixed to particle S depends not only on the forces acting on particle A, but also on the forces acting on particle S. That is, particle A can have an acceleration relative to the reference frame S even if there is no force acting on particle A, and also particle A can have an acceleration of zero (state of rest or of uniform linear motion) relative to the reference frame S even if there is an unbalanced force acting on particle A.

Finally, from the last equation above it follows that Newton's first and second laws are valid in the reference frame S only if the sum of the forces acting on particle S is equal to zero. Therefore, the reference frame S is an inertial reference frame if the sum of the forces acting on particle S is equal to zero, but it is a non-inertial reference frame if the sum of the forces acting on particle S is not equal to zero.

Bibliography

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Appendix

Dynamical Behavior of Particles

The behavior of two particles A and B which follow Newton's second law is determined from a reference frame S (inertial) by the equations:

$$\sum \mathbf{F}_{\mathrm{A}} = m_{\mathrm{A}} \mathbf{a}_{\mathrm{A}} \tag{1}$$

$$\sum \mathbf{F}_{\mathrm{B}} = m_{\mathrm{B}} \mathbf{a}_{\mathrm{B}} \tag{2}$$

that is

$$\frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \mathbf{a}_{\mathrm{A}} = 0 \tag{3}$$

$$\frac{\sum \mathbf{F}_{\mathrm{B}}}{m_{\mathrm{B}}} - \mathbf{a}_{\mathrm{B}} = 0 \tag{4}$$

Combining the equations (3) and (4) yields

$$\frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \mathbf{a}_{\mathrm{A}} = \frac{\sum \mathbf{F}_{\mathrm{B}}}{m_{\mathrm{B}}} - \mathbf{a}_{\mathrm{B}} \tag{5}$$

Therefore, the behavior of particles A and B is now determined from the reference frame S by the equation (5).

Now, if the equation (5) is transformed from the reference frame S to another non-rotating reference frame S' (inertial or non-inertial) using the transformations of kinematics and dynamics: ($\mathbf{a}' = \mathbf{a} - \mathbf{a}_{o'}$), ($\mathbf{F}' = \mathbf{F}$) and ($m' = m$), it follows that

$$\frac{\sum \mathbf{F}_{\mathrm{A}}{}'}{m_{\mathrm{A}}{}'} - \mathbf{a}_{\mathrm{A}}{}' = \frac{\sum \mathbf{F}_{\mathrm{B}}{}'}{m_{\mathrm{B}}{}'} - \mathbf{a}_{\mathrm{B}}{}' \tag{6}$$

Considering that the equation (6) have the same form as the equation (5), then it can be assumed that the behavior of particles A and B is determined from any non-rotating reference frame (inertial or non-inertial) by the equation (5).

Now, if the equation (5) is applied to a particle A and a non-rotating reference frame S (inertial or non-inertial) fixed to a particle S, then

$$\frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \mathbf{a}_{\mathrm{A}} = \frac{\sum \mathbf{F}_{\mathrm{S}}}{m_{\mathrm{S}}} - \mathbf{a}_{\mathrm{S}} \tag{7}$$

Since the acceleration $\mathbf{a}_{\mathrm{S}}$ of particle S relative to the non-rotating reference frame S equals zero always, $\mathbf{a}_{\mathrm{A}}$ may be obtained from the equation (7) as follows:

$$\mathbf{a}_{\mathrm{A}} = \frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \frac{\sum \mathbf{F}_{\mathrm{S}}}{m_{\mathrm{S}}} \tag{8}$$

Finally we obtain the equation (8) which represents the new equation of motion for a particle in classical mechanics, which can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces.